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Determining Values Using Options Contracts

DOI: 10.1515/ajle-2015-0032

Abstract: This paper presents a simple mechanism for allocating a single good in partnership dissolution. Since in the areas of probate, family, partnership and bankruptcy law the establishment of the value of assets is essential this mechanism can be useful. We illustrate its application for the case of determining the value of a house in a divorce proceeding. We show that when transactions costs are permitted, our mechanism shows a wider range of equilibrium outcomes than existing mechanisms such as Texas shootouts, implying it offers more equal and fair divisions. In addition, if private valuations of an asset are allowed, the proposed mechanism has an advantage based on the efficiency criterion compared to Texas shootouts.

Keywords: conflict; divorce; options.

1 Introduction

Determining the value of property or assets is fundamental to law. Without the establishment of value, the issues surrounding the division of assets are ambiguous.1 Some examples where the establishment of value is critical for practical applications are in the areas of probate, family law, partnership and bankruptcy law. In each of these areas, the law specifies how a particular division

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1 There is a literature which questions the commodification of certain assets and by implication their marketability or transferability. This inability would imply that value is difficult or impossible to assess. Equally, there exists the philosophical question of commensurability of commodities and therefore the inability to compare values (e.g. Sunstein 1994). For a critique of market valuation, see Radin (1987). The assumption in this paper is that choices or opportunities can be compared by individuals and that a well-defined ordering of their preferences exists.

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of the assets is to be conducted. Probate law deals how the assets from an estate are to be distributed. Family law specifies how those from a dissolved marriage are divided, while partnership law how the assets from a dissolved partnership are parceled. In bankruptcy law, the question of the apportionment of assets of a debtor to creditors is analyzed. Yet, in all these areas we must have some measure of value or worth of the disputed assets for the process of dividing or apportionment to commence. In other words, the establishment of value is a necessary, though not sufficient condition, for a successful resolution of conflicting claims to property or assets. Without the establishment of value not only would disputants be arguing over their share of the assets, but over the actual worth of these as well.

This paper is meant to address the problem of the establishment of worth or value of assets in a partnership dissolution. More specifically, let us motivate the problem with the use of an example. Suppose that two parties are in the process of a divorce and there is a dispute over the value of an indivisible asset owned by the two individuals, for example, a home. In this case it may not be practical for either party to use the market to establish price, since in conflictual situations value generally has to be determined fairly quickly to resolve the dispute and market offers occur over time. Another important reason not to place the asset on the market to determine its value is that using the market to sell the asset incurs transaction costs (e.g. realtor commission).

The objective is to find an optimal manner to establish the asset’s value under these constraints. An important requirement of any procedure is that ex ante both parties should perceive that they are better off after the transaction is completed than before they proceeded, i.e. it should result in a Pareto-improving allocation. Furthermore, the social cost of negotiating or transacting should be kept to a minimum. Ideally, both parties would be able to use the procedure without the need for court intervention or arbitration.

Continuing with the previous example, suppose that the value of a house is to be determined so that the share of the equity of one of the parties can be distributed to them. How do we assess the value of the home? A common procedure

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2 There is a literature which tries to derive methods of “fairly” dividing assets. For example, see Brams and Taylor (1996) for a discussion of proposed methods.

3 In general, as a matter of equitable distribution decisions on residential real estate involves selling the property and dividing the proceeds or having one party “buy-out” the other. The equitable distribution statutes in the US generally authorize the court to distribute the real property that was acquired by the spouses during marriage (see e.g. N.J.S.A. 2A:34-23). With the advent of pre-marital agreements, “the case law recognizes that parties may wish to contract with one another concerning the eventual distribution of property upon divorce” (N.J.S.A. 2A:34-23-1e).
is for each party to hire an appraiser and then to choose the average of the two values or if there is a large discrepancy in values between the first estimates to appoint a third appraiser as the arbiter. Another method used in practice is for only one appraiser to be chosen (supposedly agreed to by both parties in the dispute) with both parties agreeing to the established value as final. However, these two methods are flawed for several reasons. First, the average of the two values or that chosen independently by a single appraiser is not necessarily the market value as perceived by one or both of the parties. Therefore, generally one or both will feel that the established value made them worse-off than had the value been determined by some other procedure. Furthermore, one or the other party may be distrustful of the appraiser chosen by the other feeling that the estimated value assessed by the other’s appraiser is biased against them. How can we remedy the situation so that the market value is assessed without any party having an incentive to employ a biased appraiser?

We propose the following solution. Each party hires their own appraiser and proposes a price independently. The person who provides the lower estimate has the obligation to sell the house to the other party for this value. Likewise, the person who provides the higher estimate has the obligation to buy the house at this price. Flip a coin to decide which option to exercise. By writing these contracts into the procedure we are able to eliminate any incentive either party has to misinform the other of the market price, because one or the other party could either buy or sell the house for a profit. The only situation where neither party has any profit opportunity is when each hires an appraiser who values the asset at the market value. At this point the option (either “put” or “call”) is worth nothing and therefore will not be exercised. So long as the market value is not being assessed by one or the other party the option is in the money and will be exercised leaving one of the parties aggrieved.

The rest of this study proceeds in three sections. First, we review the existing research on dissolving a partnership. Next, this study presents the equilibrium analysis of the proposed mechanism based on three cases: a baseline case, a case with transaction costs, and a case with private valuations. Finally, we provide a concluding remark.

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4 “The court requires all parties and their attorneys to cooperate with the court in its endeavor to value property.” [NJ Rothman v. Rothman 65.NJ 219, 232–233 (1974)]. “The parties ... have the primary obligation of adducing those proofs which will enable the judge to make sound and rational valuations ... [T]he parties must fully cooperate in the court’s different valuation task and, where necessary, must secure the assistance of appropriate experts;” quoted in Torres v. Schripps, Inc.. supra 342 N.J. Super 419, 435–436 (App. Div. 2001). Furthermore, the parties are encouraged to stipulate values, where possible, and a trial court will be allowed to deviate from such a stipulation only where such deviation is made upon specific findings of fact and legal conclusions.” [Esposito v Esposito, 158 N.J. Super 285, 290 (App. Div. 1978)].
2 Existing Literature on Partnership Dissolution

Many existing studies have devoted attention to how to dissolve a partnership. These studies either evaluate existing mechanisms or procedures using important criteria such as fairness, efficiency, and equal-treatment, or devise new procedures to make an improvement over the existing ones. In this section, we review a number of important works on partnership dissolution.

In a series of articles, Vincent Crawford examines the properties of the “divide-and-choose” method (Crawford 1977, 1979, 1980). Based on this method, one player divides a good into two halves and the other player chooses one half. Crawford (1977) shows this method generates fair but not necessarily Pareto-efficient and equal allocations. In addition, the divider has an advantage. However, when two players have identical preferences, Pareto-efficiency and equal treatment are both achieved. To improve upon the divide-and-choose method, Crawford devises a “Pareto-efficient, egalitarian-equivalent allocations” procedure in Crawford (1979) and an “equal-division divide-and-choose” method in Crawford (1980). In the former procedure, players bid for the role of divider and the highest bidder wins it. In the latter method, the randomly selected divider offers the chooser a choice between an allocation determined by the divider and an equal division. Both mechanisms achieve Pareto-efficiency.

Subsequent studies generalize the analysis of partnership dissolution to allow for private information of players’ valuations of an asset. In a seminal work, Cramton, Gibbons, and Klemperer (1987) assume symmetric, independent private valuations and analyze a $k$-double auction mechanism. They demonstrate that when players hold equal initial shares, a partnership can always be dissolved efficiently. McAfee (1992) assumes independent private valuations and investigates four simple mechanisms: the winner’s bid auction, the loser’s bid auction, the cake-cutting mechanism, and the alternating selection mechanism. It is shown efficiency is not achieved in some of these mechanisms. Several articles examine interdependent private valuations (Fieseler, Kittsteiner, and Moldovanu 2003; Kittsteiner 2003; Jehiel and Pauzner 2006). They demonstrate that when private valuations are interdependent, it becomes difficult to dissolve a partnership efficiently – a result that contrasts with the important finding in Cramton, Gibbons, and Klemperer (1987).

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5 Ensthaler, Giebe, and Li (2014) show that when Cramton, Gibbons and Klemperer’s (1987) model is extended by introducing a proposal stage, the model no longer generates ex post efficient allocations.
A few articles analyze Texas shootouts or a shotgun clause in a private valuation setting. Similar to the divide-and-choose method, in Texas shootouts one player names a price and the other player decides whether to buy or sell at the named price. McAfee (1992) demonstrates this mechanism is ex post inefficient and it provides a chooser advantage. de Frutos and Kittsteiner (2008) argue that negotiating the role of proposer is critical for efficiency and show two cases where players negotiate this role. When private valuations are independent, a shotgun clause leads to an efficient partnership dissolution. If private valuations are interdependent, while it does not achieve full efficiency, this mechanism still performs better than other mechanisms. Brooks, Landeo, and Spier (2010) investigate why a nonmandatory Texas shootout clause is rarely triggered. They show theoretically and experimentally that players prefer simple offers to buy or to sell. When triggered, however, Texas shootouts are efficient. Assuming one-sided asymmetric information, Landeo and Spier (2013) provide theoretical and experimental evidence that when the role of offeror is designated to the informed player, shotgun mechanisms generate equal and efficient allocations.

In this study, we make a close comparison between our proposed mechanism and Texas shootouts. It generates same equilibrium allocations as Texas shootouts in the baseline case where two parties have identical valuations and these valuations are public knowledge. When transactions costs are permitted, our mechanism offers more equal and fair divisions of the gains than Texas shootouts. If private valuations are allowed, the proposed mechanism has an advantage based on the efficiency criterion.

3 A Mechanism for Dissolving Partnership

The formal statement of our proposed mechanism is the following. First, each agent reports a price $v_i$, where $i = A, B$. If $v_A \leq v_B$, party $A$ writes an option contract that gives party $B$ the option to purchase the house at the price of $v_A$, and party $B$ writes an option that allows $A$ to sell the house to $B$ at the price of $v_B$. By symmetry, if $v_A \geq v_B$, party $B$ writes an option contract that gives party $A$ the option to

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6 When preferences are risk neutral, our proposed mechanism generates identical equilibrium results as a “$k+1$ price auction” with $k = \frac{1}{2}$, an auction that has been discussed in Cramton, Gibbons and Klemperer (1987). We thank one reviewer for pointing out this similarity. However, these two mechanisms show different equilibrium outcomes when preferences are not risk neutral. The analysis for non-risk neutral preferences is beyond the scope of this study, and it can be further explored in a future study.

7 When $v_A = v_B$, one party is randomly selected to buy the house.
purchase the house at the price of \( v_B \), and party \( A \) writes an option that allows \( B \) to sell the house to \( A \) at the price of \( v_A \). One of the two options is randomly selected (e.g. flip a coin) and exercised.

### 3.1 Baseline Case

Before we present more interesting applications of our mechanism, it is useful to show a baseline case. In this baseline case, two parties \( A \) and \( B \) need to divide an indivisible asset (e.g. a property) in a divorce. Each party is indifferent between receiving the house or the money. Assume the property to be divided has a common value \( v \) to both parties \( A \) and \( B \). In addition, it is public knowledge that each party values the property at \( v \). In the next two sections, we will relax both assumptions. By introducing a transaction cost for one party, the first extension allows party \( A \) and party \( B \) to have different valuations of the property. In the second extension, we further relax the assumption of complete information, and allow each party’s valuation to be private information.

For this baseline case, our mechanism has a unique equilibrium: both party \( A \) and party \( B \) offer the price equal to his or her valuation, that is \( v_A^* = v_B^* = v \). In equilibrium, each party has a 50% chance to own the house and has a payoff equal to \( \frac{v}{2} \). The proof of equilibrium is shown in Appendix A. Since this is the simplest case, our mechanism does not show any surprising finding. In this baseline case, the proposed mechanism generates the identical equilibrium result as Texas shootouts.

### 3.2 Extension One: with Transaction Costs

In this section, we present a more interesting case, by relaxing the assumption that both parties have common valuations of the asset. Suppose now one party does not want to retain ownership. Without loss of generality, assume that party \( A \) is still indifferent between receiving the house or the money. On the other hand, party \( B \) does not want to own the house. If \( B \) owns the house, she will sell it on the market. Since selling a property on the market incurs a transaction cost \( c_B \) (e.g. closing cost calculated at 6% of the sale price), party \( B \)’s new valuation of the house becomes \( v - c_B \). In this case, party \( A \) and party \( B \) hold different valuations of the property.

For this new case with transaction costs, the equilibrium of our mechanism is as follows. There is a continuum of equilibria in the interval \([v - c_B, v]\). Each equilibrium consists of \( v_A^* \in (v - c_B, v) \) and \( v_B^* = v_A^* - \epsilon \in [v - c_B, v) \), where \( \epsilon \) is a positive...
number close to zero. In all equilibria, party $A$ purchases the house, with a 50% chance at the price $v_A^*$ and a 50% chance at $v_B^*$. Party $B$ receives half of the purchasing price, which is $\frac{v_A^*}{2}$ or $\frac{v_B^*}{2}$. In short, our mechanism indicates that when there is a transaction cost for $B$, the house can be sold at any price between $v-c_B$ and $v$. This equilibrium result is quite intuitive, since this is the range of prices that offer both party $A$ and party $B$ as least as much as their reservation values. Once again, we provide the equilibrium proof in Appendix B.

This equilibrium result implies a welfare improvement for both parties when compared to selling the house on the market. When the house is sold on the market, each party receives $\frac{v-c_B}{2}$ after subtracting the transaction cost $c_B$. On the other hand, based on our mechanism, party $A$ and party $B$ save the transaction cost $c_B$ and divide it between themselves. For party $B$, the worse expected offer she may receive is $\frac{v-c_B+\epsilon}{4}$, which is still larger than $\frac{v-c_B}{2}$. For party $A$, the minimum he may receive in expectation is $\frac{v+\epsilon}{2}$, which is much higher than $\frac{v-c_B}{2}$. In conclusion, our mechanism is a strict Pareto improvement over selling the house on the market.

When transaction costs are introduced, our mechanism offers a wider range of equilibrium results than Texas shootouts. In Texas shootouts, when party $B$ is selected to propose a price, she proposes $v_B^* = v - \epsilon$ and party $A$ chooses to buy the house. If party $A$ is chosen to offer a price, he proposes $v_A^* = v - c_B + \epsilon$ and party $B$ chooses to sell the house. When one party is randomly selected to propose a price, the expected payoff is $\frac{v + c_B}{2}$ for party $A$ and $\frac{v - c_B}{2}$ for party $B$. In short, the equilibrium offers in Texas shootouts correspond to the upper and lower bounds of the equilibrium offers indicated by our proposed mechanism. Put differently, offers within these two extreme values are not permitted in Texas shootouts.

An important feature of our mechanism in the presence of transaction costs is that it permits a multiplicity of equilibrium outcomes. We argue this is a desirable property. The above comparison between Texas shootouts and our mechanism serves as a good illustration. Based on the proposed mechanism, any price between $v-c_B$ and $v$ is a possible equilibrium offer, which implies a more equal and fair division of the gains than Texas shootouts. For example, suppose $v = \$500,000$ and $c_B = \$50,000$. In this case, parties $A$ and $B$’s reservation values are $\$250,000$ and $\$225,000$, respectively. In other words, the remaining $\$25,000$ are the gains to be divided between the two parties. When dividing the gains, the most equal method is that each party receives $\$12,500$, and the most unequal method is that one party receives everything. Texas shootout is an example of the latter that allocates the entire gains to the price namer. On the other hand, our
mechanism divides the gains between two parties in a more equal and fair way, which allows divisions that move away from one party holding all gains towards each party receiving $12,500.

3.3 Extension Two: With Incomplete Information

Since a great deal of existing research on partnership dissolution assumes incomplete information on valuations of the asset, this study further relaxes the assumption of complete information indicated in the baseline case and in the case with transaction costs. This extension is an important application of our proposed mechanism. When valuations of the house by both parties are public knowledge, our mechanism does not serve to reveal private information. Since our goal is not to fully characterize the proposed mechanism in the case of private information, in this section we present a case of symmetric and independent private valuations of the property. In addition, this study assumes each party’s private valuation is given by a uniform distribution. In conclusion, in this current extension we present a case with different valuations of the asset by two parties and these valuations are private information. In the following discussion, we will show that our proposed mechanism reveals both parties’ valuations of the house in equilibrium.

Once again, we show the proof of a unique equilibrium in Appendix C. In equilibrium, party A offers $v_A = \frac{2}{3} \tilde{v}_A + \frac{1}{6}$, and party B offers $v_B = \frac{2}{3} \tilde{v}_B + \frac{1}{6}$, where $\tilde{v}_A$ and $\tilde{v}_B$ are A and B’s true valuations of the asset. When $\tilde{v}_A > \tilde{v}_B$, party A purchases the house, with a 50% chance at the price $v_A$ and a 50% chance at $v_B$. Party B receives half of the purchasing price, which is $\frac{v_A}{2}$ or $\frac{v_B}{2}$. Likewise, when $\tilde{v}_B > \tilde{v}_A$, party B buys the house at the price $v_A$ or $v_B$, each price with $\frac{1}{2}$ probability. Party A receives half of the purchasing price at $\frac{v_A}{2}$ or $\frac{v_B}{2}$. As can be clearly seen from the equilibrium offers $v_A$ and $v_B$, our mechanism reveals private information on valuations of the asset by both party A and party B. This mechanism is also efficient, since the party that has higher valuation of the asset will be purchasing it.

As usual, we present the results based on Texas shootouts as a comparison. The same setup is applied, which assumes independent private valuations drawn from a standard uniform distribution. In equilibrium, the price namer offers $v_i = \frac{1}{2} \tilde{v}_i + \frac{1}{4}$, where $\tilde{v}_i$ is party $i$’s true valuation of the asset, and $i=A, B$. Without loss of generality, suppose A is the price namer and B decides whether to buy or to sell the property. As a result, party A’s equilibrium offer is $v_A = \frac{1}{2} \tilde{v}_A + \frac{1}{4}$, and party...
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B chooses to buy if $\tilde{v}_B^* > v_A^*$ and to sell if $\tilde{v}_B^* < v_A^*$. While Texas shootouts also reveals private information on valuations of the asset, it is not always efficient in the sense the party with higher valuation does not necessarily purchase the house. When $A$’s valuation of the property is larger than $\frac{1}{2}$, it is possible that $B$ has a lower valuation (i.e. $\tilde{v}_B < \tilde{v}_A$) but will be purchasing the house. For example, if $\tilde{v}_A = 0.75$, $\tilde{v}_B = 0.65$, we have $\tilde{v}_B^* = v_A = \frac{1}{5} \times 0.75 + 0.25 = 0.625$, which suggests $B$ will choose to buy the property. This comparison between our proposed mechanism and Texas shootouts demonstrates that our mechanism has an advantage based on the efficiency criterion.

4 Conclusion

In this paper, we propose a tractable mechanism to allocate an indivisible asset in a partnership dissolution. In this procedure, each party submits a price. The person who provides the lower price has the obligation to sell the asset to the other party for this value. Likewise, the person who provides the higher price has the obligation to buy the asset at this price. Flip a coin to decide which option to exercise.

We investigate three cases: a baseline case where two parties have identical valuations and these valuations are public knowledge, a case with transaction costs, and a case with private information of valuations. The analysis shows that when transaction costs are introduced, our mechanism permits a wider range of equilibrium allocation outcomes than existing mechanisms such as Texas shootouts. This result suggests that the proposed mechanism divides the gains more equally and fairly. In addition, if private valuations of the asset are allowed, the proposed mechanism has an advantage based on the efficiency criterion.

Appendix A

Proof for Baseline Case

In this appendix, we show that $v_A^* = v_B^* = v$ is a Nash equilibrium and this equilibrium is unique.
Suppose player $i$'s offer is $v_i = v$, that is he offers a price equal to his valuation. Given player $i$'s choice, there are three possible ranges of prices player $j$ can offer: prices higher than $i$'s offer, prices lower than $i$'s, or the price equal to $i$'s. Let us use $v_j = v_i + \epsilon$, $v_j = v_i - \epsilon$, and $v_j = v_i$ to represent these three cases, where $\epsilon$ is an arbitrary positive number. It is straightforward to show that $v_j = v_i$ gives $j$ the highest expected payoff.

First, when player $j$ offers prices higher than $i$'s offer, that is $v_j = v_i + \epsilon$. Our mechanism indicates that $j$ will buy the house at the price $v_j$ with a 50% chance and at $v_i$ with a 50% chance. Player $i$ will receive half of the purchasing price from $j$, which is $\frac{v_j}{2}$ in the former case and $\frac{v_i}{2}$ in the latter case. As a result, the expected payoff for $j$ is

$$
\frac{1}{2} \left( v_j - \frac{v_i}{2} \right) + \frac{1}{2} \left( v_i - \frac{v_j}{2} \right) = \frac{v - \epsilon}{2} - 4
$$

(A-1)

after we plug in $v_j = v_i + \epsilon$ and $v_i = v$. Since this expected payoff is decreasing in $\epsilon$, it implies that $\epsilon$ will be a value as small as possible.

When player $j$'s offers a price lower than $i$'s price, $v_j = v_i - \epsilon$. In this case, player $i$ will be purchasing the house, and $j$ will receive half of the buying price, which is either $\frac{v_j}{2}$ or $\frac{v_i}{2}$. The expected payoff for player $j$ is

$$
\frac{1}{2} \left( v_j - \frac{v_i}{2} \right) + \frac{1}{2} \left( v_i - \frac{v_j}{2} \right) = \frac{v - \epsilon}{2} - 4
$$

(A-2)

Finally, if player $j$ offers the same price as player $i$, that is $v_j = v_i$. In this case, player $j$ has a 50% chance to buy the house at the price $v_j$ and a 50% chance to receive $\frac{v_i}{2}$ from player $i$. Player $j$’s expected payoff is

$$
\frac{1}{2} \left( v_j - \frac{v_i}{2} \right) + \frac{1}{2} \left( v_i - \frac{v_j}{2} \right) = \frac{v}{2}
$$

(A-3)

As is evident, when player $i$ offers $v_i = v$, the best response for player $j$ is to offer $v_j = v_i = v$. By symmetry, when $v_j = v$, the best response for $i$ is to offer $v_i = v_j = v$. Since this is a mutual best response, $v_A^* = v_B^* = v$ is a Nash equilibrium for our proposed mechanism for the baseline case.

Next, this study demonstrates that the above equilibrium is unique. Toward this goal, we need to consider two cases.
Case one: suppose player $i$’s offer is $v_i > v$. Once again, we calculate player $j$’s expected payoffs for the three cases $v_j = v_i + \epsilon$, $v_j = v_i - \epsilon$, and $v_j = v$, and the expected payoffs are

$$\frac{1}{2} \left( v_i - \frac{\epsilon}{2} \right) + \frac{1}{2} \left( v_i - \frac{\epsilon}{2} \right) = v - \frac{v_i - \epsilon}{4}$$  \hspace{1cm} (A-4)$$

$$\frac{1}{2} \left( v_j + \frac{\epsilon}{2} \right) + \frac{1}{2} \left( v_j + \frac{\epsilon}{2} \right) = \frac{v_i + \epsilon}{2}$$  \hspace{1cm} (A-5)$$

$$\frac{1}{2} \left( v_j - \frac{\epsilon}{2} \right) + \frac{1}{2} \left( v_j - \frac{\epsilon}{2} \right) = \frac{v}{2}$$  \hspace{1cm} (A-6)$$

When we compare $j$’s expected payoffs from the above three cases, we have $v - \frac{v_i - \epsilon}{2} < v$ given $v < v_i$ and $\epsilon > 0$. In addition, $\frac{v_i + \epsilon}{2} < v$ since an optimal value of $\epsilon$ is chosen to be close to zero. Therefore, when player $i$ offers a price higher than his own valuation, the best response for player $j$ is to choose $v_j = v_i - \epsilon$. By symmetry, when $j$ chooses $v_j > v$, the best response for player $i$ is $v_i = v_j - \epsilon$. In conclusion, no player will offer an equilibrium price higher than his or her value.

Case two: suppose player $i$’s offer is $v_i < v$. In this case, the calculations of player $j$’s expected payoffs are identical to case one. For $v_j = v_i + \epsilon$, $v_j = v_i - \epsilon$, and $v_j = v$, the respective expected payoffs are

$$\frac{1}{2} \left( v_i + \frac{\epsilon}{2} \right) + \frac{1}{2} \left( v_i + \frac{\epsilon}{2} \right) = v - \frac{v_i + \epsilon}{4}$$  \hspace{1cm} (A-7)$$

$$\frac{1}{2} \left( v_j - \frac{\epsilon}{2} \right) + \frac{1}{2} \left( v_j - \frac{\epsilon}{2} \right) = \frac{v_i - \epsilon}{2}$$  \hspace{1cm} (A-8)$$

$$\frac{1}{2} \left( v_j - \frac{\epsilon}{2} \right) + \frac{1}{2} \left( v_j - \frac{\epsilon}{2} \right) = \frac{v}{2}$$  \hspace{1cm} (A-9)$$

Because $v_i < v$, we have $\frac{v_i - \epsilon}{2} + \frac{\epsilon}{2} < v$. Furthermore, $\frac{v_i + \epsilon}{2} < v - \frac{v_i + \epsilon}{4}$ since equation (A-7) is maximized when $\epsilon$ is close to zero. As a result, in case two the best response for $j$ is $v_j = v_i + \epsilon$. By symmetry, when $j$ chooses $v_j < v$, the best response for player $i$ is $v_i = v_j + \epsilon$. In conclusion, no player will offer an equilibrium price lower than $v$. 

Appendix B

Proof for the Case of Transaction Costs

In appendix B, we show the equilibrium proof for the case with transaction costs. Since party A has a value of \( v \) for the property and is indifferent between receiving the house or the money, the equilibrium analysis from the baseline case still holds for party A. That is, A’s best responses are \( v^*_A = v^*_B - \epsilon \) if \( v^*_B > v \), \( v^*_A = v^*_B + \epsilon \) if \( v^*_B < v \), and \( v^*_A = v^*_B \) if \( v^*_B = v \). However, we need to modify party B’s expected payoffs and best responses given that she pays \( c_B \) to sell the house on the market if she owns it. The equilibrium analysis is comprised of several cases.

Case one: when player A offers \( v^*_A = v \). For player B, she has three possible ranges of prices to offer: \( v^*_B = v^*_A + \epsilon \), \( v^*_B = v^*_A - \epsilon \), and \( v^*_B = v^*_A \). Their respective expected payoffs are

\[
\frac{1}{2} \left( v - \frac{v^*_B - c_B}{2} \right) + \frac{1}{2} \left( v - \frac{v^*_A - c_B}{2} \right) = \frac{v - \epsilon}{4} c_B
\]

(B-1)

\[
\frac{1}{2} \left( v^*_B - \frac{v}{2} \right) + \frac{1}{2} \left( v^*_A - \frac{v}{2} \right) = \frac{v - \epsilon}{4}
\]

(B-2)

\[
\frac{1}{2} \left( v^*_B - \frac{v^*_A - c_B}{2} \right) + \frac{1}{2} \left( v^*_B - \frac{v^*_A - c_B}{2} \right) = \frac{v - c_B}{2}
\]

(B-3)

Given that a smallest possible positive value of \( \epsilon \) maximizes equations (B-1) and (B-2), \( \epsilon \) is chosen to be close to zero. This result suggests that \( v^*_B = v^*_A + \epsilon = v - \epsilon \). Therefore, when \( v^*_A = v \), player B’s best response is \( v^*_B = v - \epsilon = v - \epsilon \). Meanwhile, given \( v^*_B < v \), player A’s best response is \( v^*_A = v^*_B + \epsilon = v \). In conclusion, since \( v^*_A = v \) and \( v^*_B = v - \epsilon \) are mutual best responses, this is a Nash equilibrium.

Case two: assume player A’s offer is \( v^*_A < v \). Once again, for player B we calculate the expected payoffs for \( v^*_B = v_A + \epsilon \), \( v^*_B = v_A - \epsilon \), and \( v^*_B = v_A \), which are

\[
\frac{1}{2} \left( v - \frac{v^*_B - c_B}{2} \right) + \frac{1}{2} \left( v - \frac{v^*_A - c_B}{2} \right) = v - \frac{v - \epsilon}{4} c_B
\]

(B-4)

\[
\frac{1}{2} \left( v^*_B - \frac{v}{2} \right) + \frac{1}{2} \left( v^*_A - \frac{v}{2} \right) = \frac{v - \epsilon}{4}
\]

(B-5)
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\[
\frac{1}{2} \left( v - \frac{v_B - c_B}{2} \right) + \frac{1}{2} \left( \frac{v_B}{2} \right) = v - \frac{c_B}{2}
\]

(B-6)

In this case, in order to find B’s best response, we need to examine several sub-cases.

Sub-case one: if \( v_A > v - c_B \). This condition implies that

\[
\left( \frac{v_A}{2} - \frac{\epsilon}{4} \right) - \left( v - \frac{v_A}{2} - \frac{\epsilon}{4} - c_B \right) = v_A - v + c_B > 0
\]

(B-7)

\[
\left( \frac{v_A}{2} - \frac{\epsilon}{4} \right) - \left( \frac{v}{2} - \frac{c_B}{2} \right) = \frac{1}{2} (v_A - v + c_B) - \frac{\epsilon}{4} > 0
\]

(B-8)

Equations (B-7) and (B-8) suggest that player B’s best response is to choose \( v^*_B = v_A - \epsilon \). In addition, since it is assumed \( v_A < v \), it follows that \( v^*_B = v_A - \epsilon < v \), suggesting that player A’s best response is \( v^*_A = v_B + \epsilon \). Because \( v^*_B = v^*_A - \epsilon \) and \( v^*_A = v^*_B + \epsilon \) are mutual best responses, we have a continuum of Nash equilibria for \( v_A \in (v - c_B, v) \).

Sub-case two: if \( v_A = v - c_B \). This assumption leads to

\[
\left( \frac{v_A}{2} - \frac{\epsilon}{4} \right) - \left( v - \frac{v_A}{2} - \frac{\epsilon}{4} - c_B \right) = v_A - v + c_B = 0
\]

(B-9)

\[
\left( \frac{v_A}{2} - \frac{ \epsilon }{4} \right) - \left( \frac{v}{2} - \frac{c_B}{2} \right) = \frac{1}{2} (v_A - v + c_B) - \frac{\epsilon}{4} < 0
\]

(B-10)

As a result, player B’s best response in this case is \( v^*_B = v_A = v - c_B \). Meanwhile, given \( v^*_B < v \), player A’s best response is \( v^*_A = v^*_B + \epsilon = v - c_B + \epsilon \neq v - c_B \). Since A has an incentive to deviate, A will not offer \( v_A = v - c_B \) in equilibrium.

Sub-case three: if \( v_A < v - c_B \). We have

\[
\left( v - \frac{v_A}{2} - \frac{\epsilon}{4} - c_B \right) - \left( \frac{v_A}{2} - \frac{c_B}{2} \right) = \frac{1}{2} (v - v_A - c_B) - \frac{\epsilon}{4} > 0
\]

(B-11)

\[
\left( v - \frac{v_A}{2} - \frac{\epsilon}{4} - c_B \right) - \left( \frac{v_A}{2} - \frac{c_B}{2} \right) = v - v_A - c_B > 0
\]

(B-12)

Therefore, player B’s best response is \( v^*_B = v_A + \epsilon \). Moreover, player A’s best response is \( v^*_A = v_B + \epsilon \) when \( v_B < v \). In conclusion, neither player will offer an equilibrium price lower than \( v - c_B \).
Case three: assume player A’s offer is $v_A > v$. As usual, for $v_B = v_A + \epsilon$, $v_A = v_A - \epsilon$, and $v_B = v_A$, player B’s expected payoffs are

$$
\frac{1}{2} \left( v_B - \frac{v_B}{2} - c_B \right) + \frac{1}{2} \left( v_A - \frac{v_A}{2} - c_B \right) = v_A - \frac{\epsilon}{4} - c_B
$$

(B-13)

$$
\frac{1}{2} \left( \frac{v_B}{2} \right) + \frac{1}{2} \left( \frac{v_A}{2} \right) = v_A - \frac{\epsilon}{4}
$$

(B-14)

$$
\frac{1}{2} \left( v_B - \frac{v_B}{2} - c_B \right) + \frac{1}{2} \left( \frac{v_B}{2} \right) = v - c_B
$$

(B-15)

Because $v_A > v$ and $\epsilon$ is chosen to be a very small number to maximize equations (B-13) and (B-14), we have $\frac{v_A - \epsilon}{2} > \frac{v_B}{2} > v - \frac{v_A - \epsilon}{2}$. As a result, player B’s best response is $v_B^* = v_A - \epsilon$. Meanwhile, when $v_B > v$, player A’s best response is $v_A^* = v_B - \epsilon$. We conclude that neither player will offer an equilibrium price higher than $v$.

**Appendix C**

**Proof for the Case of Incomplete Information**

In this section, we show the equilibrium proof for the case with private valuations. Assume parties A and B’s true valuations of the property are $\tilde{v}_A$ and $\tilde{v}_B$, respectively. Each party knows his or her own valuation, but does not know the other party’s valuation. Instead, each party only knows that the other party’s valuation of the property is given by a uniform distribution on $[0, 1]$. Let each party’s offer be a linear function of its true valuation. That is, $v_A = \alpha_A + \beta_A \tilde{v}_A$ and $v_B = \alpha_B + \beta_B \tilde{v}_B$, where $\alpha_A$, $\alpha_B$, $\beta_A$, and $\beta_B$ are constant. To solve the equilibrium in this case is to find the values for $\alpha_A$, $\alpha_B$, $\beta_A$, and $\beta_B$. Since both $\tilde{v}_A$ and $\tilde{v}_B$ follow the $[0, 1]$ uniform distribution, both parties’ offers $v_A$ and $v_B$ are given by the uniform distributions $[\alpha_A, \alpha_A + \beta_A]$ and $[\alpha_B, \alpha_B + \beta_B]$, respectively.

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8 In addition, we derive the equilibrium without assuming any functional form of each party’s offer, and show these two methods generate identical results. For illustration purpose, this study chooses to present the simpler approach.
To derive the equilibrium, we need to solve the maximization problem for each party. Without loss of generality, this study first derives party A’s best response. For A, its expected utility is

\[
E(u_A) = Pr(v_A > v_B) \left[ \frac{1}{2} \left( \tilde{v}_A - \frac{1}{2} v_A \right) + \frac{1}{2} \left( \frac{v_B - 1}{2} E(v_B | v_A > v_B) \right) \right] \\
+ Pr(v_A < v_B) \left[ \frac{1}{2} \left( \frac{1}{2} v_A \right) + \frac{1}{2} \left( \frac{1}{2} E(v_B | v_A < v_B) \right) \right] \tag{C-1}
\]

In the above equation, when A offers a higher price (i.e. \( v_A > v_B \)), A will be purchasing the property with \( \frac{1}{2} \) probability at price \( v_A \) and \( \frac{1}{2} \) probability at price \( v_B \) [i.e. \( E(v_B | v_A > v_B) \)]. If B’s offer is higher (i.e. \( v_A < v_B \)), A will receive half of the purchasing price, with \( \frac{1}{2} \) probability at price \( v_A \) and \( \frac{1}{2} \) probability at price \( v_B \) [i.e. \( E(v_B | v_A < v_B) \)].

Since \( Pr(v_A > v_B) = Pr(v_A > \alpha_B + \beta_B \tilde{v}_B) = Pr \left( \tilde{v}_B < \frac{v_A - \alpha_B}{\beta_B} \right) = \frac{v_A - \alpha_B}{\beta_B}, \quad E(v_B | v_A > v_B) = \frac{\alpha_B + v_A}{2} \), and \( E(v_B | v_A < v_B) = \frac{v_A + \alpha_B + \beta_B}{2} \), equation (C-1) becomes

\[
E(u_A) = \frac{v_A - \alpha_B}{\beta_B} \left[ \frac{1}{2} \left( \tilde{v}_A - \frac{1}{2} v_A \right) + \frac{1}{2} \left( \frac{1}{2} v_A + \frac{1}{2} \alpha_B + \frac{1}{2} \beta_B \right) \right] \\
+ \left( 1 - \frac{v_A - \alpha_B}{\beta_B} \right) \left[ \frac{1}{2} \left( \frac{1}{2} v_A \right) + \frac{1}{2} \left( \frac{1}{2} v_A + \frac{1}{2} \alpha_B + \frac{1}{2} \beta_B \right) \right] \tag{C-2}
\]

Take the first order derivative of \( E(u_A) \) w.r.t. \( v_A \), we have

\[
\frac{dE(u_A)}{dv_A} = \frac{1}{\beta_B} \left[ \frac{1}{2} \left( \tilde{v}_A - \frac{1}{2} v_A \right) + \frac{1}{2} \left( \frac{1}{2} v_A + \frac{1}{2} \alpha_B + \frac{1}{2} \beta_B \right) \right] + \frac{v_A - \alpha_B}{\beta_B} \left( -\frac{3}{8} \right) \\
+ \left( -\frac{1}{\beta_B} \right) \left[ \frac{1}{2} \left( \frac{1}{2} v_A \right) + \frac{1}{2} \left( \frac{1}{2} v_A + \frac{1}{2} \alpha_B + \frac{1}{2} \beta_B \right) \right] + \left( 1 - \frac{v_A - \alpha_B}{\beta_B} \right) \left( \frac{3}{8} \right) \tag{C-3}
\]

Let equation (C-3) equal to 0, and solve \( v_A^* \) to get party A’s best response

\[
v_A^* = \frac{2\alpha_B + \beta_B + 2\tilde{v}_A}{6} + \frac{2\tilde{v}_A}{3} \tag{C-4}
\]

As indicated in equation (C-4), A’s optimal offer \( v_A^* \) is a linear function of its valuation of the property \( \tilde{v}_A \).
Since this game is symmetric, by the same reasoning we derive the best response for $B$

$$V_B^* = \frac{2\alpha_A + \beta_A + 2\tilde{v}_B}{6}$$  \hspace{1cm} (C-5)$$

Now we can solve for $\alpha_A$, $\alpha_B$, $\beta_A$, and $\beta_B$. Since $V_A^* = \frac{2\alpha_B + \beta_B + 2\tilde{v}_A}{6}$ and $V_B^* = \alpha_A + \beta_A + \tilde{v}_A$, it follows that $\beta_A = \frac{2}{3}$ and $\alpha_A = \frac{2\alpha_B + \beta_B}{6}$. By the same token, $\beta_B = \frac{2}{3}$ and $\alpha_B = \frac{2\alpha_A + \beta_A}{6}$. It further implies that $\alpha_A = \alpha_B = \frac{1}{6}$. Therefore, parties A and B’s equilibrium offers are

$$V_A^* = \frac{1}{6} + \frac{2\tilde{v}_A}{3}$$  \hspace{1cm} (C-6)$$

$$V_B^* = \frac{1}{6} + \frac{2\tilde{v}_B}{3}$$  \hspace{1cm} (C-7)$$

References


